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THEORETICAL MECHANICS.

Theoretical Mechanics: an introductory treatise on the Principles of Dynamics, with applications and numerous examples. By A. E. H. Love, M.A., F.R.S., Fellow and lecturer of St. John's College, Cambridge. Pp. xiv + 379. (Cambridge: at the University Press, 1897.)

THIS book is vibrating with dynamical modernity, and proves in effect that Theoretical Dynamics has not yet been reduced to the level of one of the Exact Sciences; and so it shows little tendency to bridging over the gap still existing between the two modes of treatment of the one science of Mechanics. The two different methods are described by Newton in the preface of the "Principia"—

> "Auctoris præfatio ad lectorem. Cum Veteres Mechanicam (uti Auctor est Pappus) in rerum Naturalium investigatione maximi fecurunt; et Recentiores, missis formis substantialibus et qualitatibus occultis, Phænomena Naturæ ad leges Mathematicas revocare aggressi sint: Visum est in hoc Tractatu Mathesin excolere, quatenus ea ad Philosophiam spectat.
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> "Mechanicam vero duplicem Veteres constituerunt: Rationalem quæ per Demonstrationes accurate procedit, et Practicam. Ad Practicam spectant Artes omnes Manuales, a quibus utique Mechanica nomen mutuata est. Cum autem Artifices parum accurate operari solent, sit ut Mechanica omnis a Geometria ita distinguatur, ut quicquid accuratum sit ad Geometriam referatur, quicquid minus accuratum ad Mechanicam. Attamen errores non sunt Artis sed Artificum. . . .
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> "Pars hæc Mechanicæ à Veteribus in Potentiis quinque ad artes manuales spectantibus exculta fuit, qui Gravitatem (cum potentia manualis non sit) vix aliter quam in ponderibus per potentias illas movendis considerarunt. . . ."

Rankine had this preface in his mind in preparing his inaugural address (1856), a "Preliminary Dissertation on the Harmony of Theory and Practice in Mechanics," prefixed to his treatise on Applied Mechanics.

> "In physics and mechanics the notions of the Greeks were very generally pervaded by a great fallacy, which obtained its complete and most mischievous development amongst the mediæval schoolmen, and the remains of whose influence can be traced even at the present day—the fallacy of a *double system of natural laws*; one theoretical, geometrical, rational, discoverable by contemplation, applicable to celestial, ætherial, indestructible bodies, and being an object of the noble and liberal arts; the other practical, mechanical, empirical, discoverable by experience, applicable to terrestrial, gross, destructible bodies, and being an object of what were once called the vulgar and sordid arts."

We want in our theoretical treatises more of the spirit expressed on the title-page of Hayes's Fluxions, 1704, "A work very useful to those who would know how to apply Mathematics to Nature."

To do this we must come to close quarters, and "missis formis substantialibus et qualitatibus occultis" fire off the elegant artillery of analysis; in fact, reduce the formulas to their numerical applications; it is in this way only that the various differences so notable in the mode of treatment in different schools can ultimately become reconciled.

Suppose we set up our author as the champion of the first of these two schools of thought described above by Rankine, and pit him against Prof. Perry, as the champion mathematician of the engineers.

The first point of dispute will be the measurement of force; the engineer will insist on retaining in Dynamics the statical gravitational measure of force, considering that he works in a field of gravity, practically uniform over the surface of the Earth, on which the human race is imprisoned; and also because the gravitational measure of a force is the only one capable of direct experimental determination to the highest degree of accuracy; this is not the case with the absolute measure of force, the one solely adopted in the demonstrations of the present treatise.

There are certain advantages in recording the results of cosmical, electrical, magnetical, and astronomical results in absolute measure; for if the author should succeed in having his treatise adopted on another planet, his C.G.S. units would be immediately applicable, on the assumption of perfect astronomical observation and measurement; but for experimental verification each planet would have recourse to its own gravitation system.

A problem proposed recently in an American technical journal, "to find the work required to lift the Earth one foot," might perhaps serve a useful purpose in focussing discussion between the merits of absolute and gravitation measure.

A curious note on the last page of this treatise dismisses the units in which all our engineering calculations are carried out, in a few lines, such as—

> "Thus the equation which we write $P = mf$, where P is the force producing acceleration f in a body of mass m, could be written in these units $P = (m/g)f$, where g is the same constant."
>
> "It does not tend to simplicity that the writers who use these (*i.e.* the gravitation) units also use the word 'weight' for the quantity we call 'mass,' and the letter W where we use m, and thus they write the above equation $P = (W/g)f$."
>
> "Much confusion has thereby been produced."

But Prof. Perry will retort by saying that the confusion is produced by those writers who never have to employ the theory they teach; and that the words "frequently not" should be changed to "never" in the statement in § 299—

> "The C.G.S. system of units, although generally used in scientific work, is frequently not employed in practical applications of science."

Such a thing as an arithmetical mistake is unknown among those who work with gravitation units; the same cannot be said of the adherents of absolute measure who are very apt to slip a g in their calculations (there is a g missing in the result of ex. 60, p. 75.)

How does our author reconcile his definitions in Chapter v. with the precise legal terminology of the Act of Parliament on Weights and Measures?—

> "The *weight* in vacuo of the platinum *weight* (mentioned in the First Schedule to this Act), and by this Act declared to be the imperial standard for determining the imperial standard pound, shall be the legal standard measure of *weight*, and of measures having reference to

weight, and shall be called the imperial standard pound, and shall be the only unit or standard of *weight* from which all other *weights* and all measures having reference to *weight* shall be ascertained."

How does Mr. Love propose to edit this clause? The word *weight* makes its appearance seven times where Mr. Love says the right word to employ is *mass;* he cuts the Act of Parliament to pieces on p. 98; and we have mass occur in almost every line. And if the word *weight* is to go, what is to be done with *pound*, *poids* (de kilogramme), and *avoirdupois*, all derived from the Latin *pondus?* According to § 91, *pondus* is given in dynes, and the word *pondus* above must be replaced by *massa*.

If this process of Restoration (to use the banal architectural word—

> " to erect
> New buildings of correctest conformation
> And throw down old, which he called *restoration*."
> DON JUAN.)

is to be carried out systematically, what is to be done with the words "in ponderibus movendis" of Newton's preface? and how are Ovid's lines to be restored describing the statue of Ladas, the work of the sculptor Myro?—

> " Quæ nunc nomen habent operosi signa Myronis
> Pondus iners quondam duraque massa fuit."

Or again the lines—

> " . . . et gravitate carentem Æthera. . . .
> Cum quæ pressa diu massa latuere sub illa . . ."

Love's *Dynamics* versus Ovid's *Ars amatoria!* not to mention the ecclesiastical usage of Christmas, Childermass, Candlemass, Ladymass, Lammas, Loafmass, Martinmass, Soulmass, Michaelmass, . . . now exciting controversy in another place.

"The common use of the word "weight" covers two notions which are essentially distinct, the notion of pressure which a heavy body exerts on a support, and the notion of quantity of matter. In scientific writing and speaking, different words must be used to express distinct notions" (p. 99).

A very useful aphorism, worth adding to Newton's "Regulæ Philosophandi"; and so scientific writers must invent two new words to express these two distinct notions, and not attempt to force a word of common currency out of its most extended meaning.

At the same time another rule might have been made —"The names of a thing must not be multiplied more than is necessary."

"Since the centre of inertia of body small enough to be handled coincides with its centre of gravity as defined in Statics, we shall denote it by the letter G" (p. 102).

And now we have three names, *centre of mass* (d'Alembert), *centroid* (Clifford), and *centre of inertia*, where the single name centre of gravity is sufficient for ordinary purposes. It is a pity to waste the expression "centre of inertia" in this way, as it may prove useful for designating a point distinct from the centre of gravity, in the case of non-rigid systems, such as a carriage on wheels, or a fish, bird, or projectile moving in its medium.

This brings us to the "Conception of a Rigid System" in § 114—

"If the particles of a rigid system continuously fill a surface, the system is a rigid body, and the surface is the surface of the body."

At this rate the ball-bearings of a bicycle constitute a rigid system, contrary to the function for which they are designed.

The bicycle has done wonders in familiarising our youth with dynamical sensations; and the machine itself can be used in a variety of ways to illustrate the theory of the pendulum and the gyroscope. When testing the wheels for friction and balance, the elliptic functions, defined in rather a condensed way in § 191, can easily be watched in their fluctuations; while the new drawing-room game of trying to walk round holding a revolving wheel serves to emphasise gyroscopic domination. With this stimulus the languishing study of elliptic functions may again become popular, and lead on to the dynamical applications of the hyper-elliptic functions, sketched out by Prof. Klein in his Princeton lectures, as required for the complete solution of the bicycle problem, especially as the Prize offered by the French Academy for this subject is still open.

The influence of wind will excite an interest in § 212, on the motion in a resisting medium. In this article the author could have simplified the treatment, by introducing the notion of "terminal velocity," as in ex. 155, p. 227.

The statement on p. 195, that the resistance of the air is better represented by the cubic law, is not valid, except for a very limited region in the neighbourhood of the velocity of sound; but, considering that the retardation

$$\frac{d^2s}{dt^2}$$

can be replaced by

$$\frac{d^2t}{ds^2}v^3,$$

Mr. Bashforth found it convenient, in the reduction of his screen records, to take out the factor v^3, and to measure carefully the other factor, $\frac{d^2t}{ds^2}$.

The Science of Dynamics does not consist in labelling certain physical quantities with letters, such as m, W, f, g, . . .; these letters really mean numbers, expressed each in its own unit. Mathematical Tripos questions unfortunately pay scant attention to the units involved, and our mathematical students learn to loathe all numerical applications, and so lose sight of the true meaning of these algebraical symbols for numbers. One reason for this dislike of numerical computation is the absurd system of using 7 figure logarithms, where, as in the case of the gravitation constant γ, upon which all Celestial Dynamics depend, the numbers do not warrant such refinement. A gigantic cheese-auger cannot be driven into the earth, to determine the density of the strata up to the centre, so we have to be content with the indication of the Cavendish experiment, which, even in the experienced hands of Mr. Boys, do not warrant the use of logarithms of more than 4 places.

The two papers on the theory of the oscillations of a ship, and of the stresses produced thereby, read recently before the Institution of Naval Architects by Captain Kriloff, Professor at the Naval Academy of St. Petersburg, are worth the attention of theoretical students in showing the numerical computations, given to 3 significant figures only, required in a complicated problem of Rigid Dynamics, and showing also the system of gravitation units invariably employed in such calculations.

The letters *m* and W are the modern dynamical equivalents of the θ and π, the θεωρητική and πρακτική, embroidered on the hem of the robe of the vision which appeared to Boethius in his dungeon, to inspire his *Consolation of Philosophy*.

Let the letter W still continue to denote the number of pounds of matter in the body, and let *m* denote the number of grammes; let us adopt the method of Prof. T. W. Wright's "Mechanics," reviewed by Prof. Perry, a new edition of which has just appeared, and employ the absolute system with Metric units only, so that the "poundal" is merely mentioned once to point out its uselessness. Now Prof. Perry can denote $W \div 32\cdot1912$ by the letter M, so that the unit of M is a 32·1912 pound shot; and if he calls M the mass of the body, in opposition to Mr. Love, he is only following the custom which can be traced back through the treatises of Todhunter, Parkinson, Earnshaw, Whewell, Poisson, Lagrange, &c., up to Euler.

Thus M. de Freycinet writes, in his *Essais sur la philosophie des Sciences*:—

> "Il ne suffit pas d'avoir la notion claire de la masse. Il faut aller plus loin. Pour les besoins de la Dynamique il est nécessaire de savoir *chiffrer* les masses.—Une quantité d'eau peu inférieure a 10 décimètres cubes, soit 9 litres, 8088, . . . le nombre habituellement désigné par la lettre *g*, voilà l'unité de masse."

With these writers we find that the gravitation unit of force alone is employed, and, contrary to Mr. Love's classification in § 294, the unit of mass is a *derived* unit, being that quantity of matter which will receive unit acceleration from the gravitational unit of force. The same method is employed in all engineering treatises, but we are inclined to agree with Mr. Love in thinking it might be abandoned with advantage, as being a mere lazy device to avoid writing $\frac{W}{g}$; and coming back to Euler, we find him explaining at length, in some six pages of his "Dynamics," 1760, that the acceleration $a = \lambda \frac{P}{M}$, due to a force P acting on a mass M, and that we must take $\lambda = 2g$, where g is taken by Euler to measure the distance a body falls from rest in one second.

Students will be grateful to the author for the two elegant and complete chapters on two-dimensional Motion of a Rigid Body, a great desideratum. A very large and valuable collection of illustrative examples are brought together, most of which are capable of experimental verification in our field of gravity; and in such cases it would increase the instructiveness to employ the gravitation measure of force, the only one capable of exact measurement.

> "When, as in astronomy, we endeavour to ascertain (these) causes by simply watching their effects, we *observe;* when, as in our laboratories, we interfere arbitrarily with the causes or circumstances of a phenomenon, we are said to *experiment*" (Thomson and Tait).

In recording theoretical results of astronomical observation, absolute units are certainly appropriate, but they are all susceptible to the probable error in the determination of the gravitation constant γ.

The author has performed a useful service in § 277, in calling attention to the looseness of the ordinary school-book definitions, that "the weight of a body is the force with which it is attracted by the Earth."

But we must return to the charge again, and protest against the assumption that the addition of the word "weight" to "pounds" is required to connote the idea of force. Architects may measure the pressure on foundations in cwt/ft^2, but there is no such thing in existence as a pressure gauge graduated in lbs-wt/in^2; it is always in lbs/in^2; more than that, we doubt the existence of any gauge graduated in dynes/cm^2, or barads; and the stock of instruments at present in use is sufficiently large to resist this innovation. The centesimal measurement of time, required for the completeness of the metric decimal system, never came into use, if only because of the number of clocks, watches and chronometers in existence; so that the C.G.S. system is a mongrel one, involving the sexagesimal second of time.

In the careful examination of the ultimate axioms of Dynamics which he has set himself for reconsideration, the author has thrown down a challenge to the Metaphysicians, in the theory of the relativity not only of motion, of rotation as well as of translation, but also of time, matter, force, &c., which we trust will not pass unnoticed.

This minute survey of the foundations of Dynamics has, like a visit to the dentist, revealed so many unsuspected flaws, that it seems doubtful if Dynamics can remain an exact Science. Considering that the gravitation of a body varies with the velocity relative to the Earth, how are we justified in accepting the sacred definitions of the C.G.S. units, which may be affected by similar defects? A spirit of dynamical scepticism is in the air, as testified by the treatises of Mach and his disciples, Hertz, Boltzmann, and by Poincaré on Hertz in the *Revue générale des Sciences*. Maxwell's and Clifford's work does not appear to have influenced the author.

According to the Preface, "The foundations of Mechanical Science were laid by Newton"; but we think that the claims of Galileo are passed over, not to mention Archimedes. Galileo appears throughout this treatise as Galilei; both forms of the name are correct, according to the German student song—

> "Auch ging er wohl mitunter
> Zur Kirche als frumber Mann;
> Doch beten und singen nicht kunnt er,
> Schaut lieber zur Decke hinan:
> Was sah er da in der Höhi?
> Tschahi, tschaheia ho—
> Die Ampel sah Galilei
> Und auch der Galileo!" &c.

Some novelties in the way of nomenclature are welcome, such as "frame of reference" for "coordinate axes," "localised vector," "kinetic reaction" (due to Mr. Larmor, we believe) for d'Alembert's reversed effective force"; but when the writer proposes to upset the well-established use of common words, and teach us a new language of recent invention, he might as well set to work to change the names of the stars and planets; and we are compelled to protest, in the words of Biron,

> "These earthly godfathers of heaven's lights,
> That give a name to every fixed star,
> Have no more profit of their shining nights
> Than those that walk, and wot not what they are."

A. G. Greenhill.